Electroweak Parameters from $\mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$: Twenty-Three Predictions with Zero Free Parameters

Richard Astbury

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Abstract

Working within the CHO framework, where the Standard Model arises from the algebra $\mathcal{A} = \mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ equipped with an information-theoretic action on a causal lattice, we derive twenty-three electroweak and flavour observables from first principles with no free parameters. The top Yukawa coupling $y_t = 1$ follows from norm saturation (Cauchy–Schwarz on $\mathcal{A}$), giving $m_t = v/\sqrt{2} = 174.1$ GeV. The Higgs quartic $\lambda = \pi/24$ follows from the D_4 root system, giving $m_H = v\sqrt{\pi/12} = 126.0$ GeV. The fine structure constant $\alpha^{-1} = 128\pi/3 = 134.0$ at the confinement scale, running to $\alpha^{-1}(0) = 137.0$ via standard vacuum polarization. The Weinberg angle $\sin^2\theta_W = 1/4$ at the same lattice scale runs to $\sin^2\theta_W(M_Z) = 0.231$ via 1-loop SM evolution. The neutrino mass $m_{\nu_3} = v^2/(2M_R) = 48.9$ meV follows from the seesaw with $M_R = M_\mathrm{P}/3^9$, and the mass splitting ratio $\Delta m^2_{21}/\Delta m^2_{31} = 4\varepsilon_0^2$ (-0.5σ). The CP-violating Jarlskog invariant $J = 3.01 \times 10^{-5}$ follows from the NNI texture and the Fano plane angle $\delta = \arccos(1/3)$. The PMNS mixing angles $\sin^2\theta_{13} = 3\varepsilon_0^2$ (-0.4σ), $\sin^2\theta_{12} = 1/(3 + \sqrt{7}\,\varepsilon_0)$ ($+0.2\sigma$), and $\sin^2\theta_{23} = 4/7$ (-0.02σ) correct tribimaximal mixing to sub-percent accuracy. The CKM magnitudes $|V_{us}| = \sqrt{7}\,\varepsilon_0$ (0.6%) and $|V_{cb}| = \varepsilon_0/2$ (1.0%) are determined by the same triality parameter $\varepsilon_0^2 = \pi/432$. Three inter-sector mass relations with factors $N_c = 3$ and $\dim(\mathbb{O}) = 8$ provide a first-principles derivation of the Georgi–Jarlskog factor. Eight fermion masses (three generations) follow from m_t alone via the NNI texture. All 23 predictions agree with experiment at the 0.1–7% level (median 1.0%). No parameters are adjusted.

Contents

1 Introduction

The Standard Model of particle physics contains 19 free parameters whose values are determined by experiment. Among the deepest questions in fundamental physics is whether these parameters are calculable from a more fundamental structure.

We work within the CHO framework, where the physics algebra is $\mathcal{A} = \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ (64 real dimensions), spacetime is a causal lattice, and the dynamics is governed by an information-theoretic action [1–3]:

$$S = \sum_{\langle xy \rangle} \log \cos \theta_{xy}, \tag{1}$$

where θ_{xy} is the angle between labels $\phi(x), \phi(y) \in \mathcal{A}$ on adjacent lattice sites. In a companion paper [4], we proved that this algebra requires exactly three fermion generations. In a further companion [16], the same suppression mechanism yields a cosmological constant $\Lambda^{1/4} = 2.31$ meV with no free parameters.

Here we show that the *same* structure, with no additional assumptions, determines:

1. The top quark Yukawa coupling: $y_t = 1$
2. The Higgs boson mass: $m_H = v\sqrt{\pi/12}$
3. The fine structure constant: $\alpha^{-1}(0) = 137.0$
4. The Weinberg angle: $\sin^2\theta_W(M_Z) = 0.231$
5. The heaviest neutrino mass: $m_{\nu_3} \approx 49$ meV
6. The neutrino mass splitting: $\Delta m^2_{21}/\Delta m^2_{31} = 4\varepsilon_0^2$
7. The electroweak hierarchy: $v/M_\text{P} = (1/\sqrt{3})^{72}$
8. CP violation: $J_\text{CKM} = 3.01 \times 10^{-5}$
9. The CKM matrix: $|V_{us}| = \sqrt{7}\,\varepsilon_0$, $|V_{cb}| = \varepsilon_0/2$
10. PMNS mixing: $\sin^2\theta_{13} = 3\varepsilon_0^2$, $\sin^2\theta_{12} = 1/(3+\sqrt{7}\varepsilon_0)$, $\sin^2\theta_{23} = 4/7$
11. Proton stability: $\tau_p > 10^{64}$ years

The logical chain is:

$$M_\text{P} \xrightarrow{(1/\sqrt{3})^{72}} v \xrightarrow{y_t=1} m_t \xrightarrow{\lambda=\pi/24} m_H \tag{2}$$

$$M_\text{P} \xrightarrow{(1/\sqrt{3})^{18}} M_R \xrightarrow{\text{see-saw}} m_{\nu_3} \tag{3}$$

The only measured input is $M_\text{P} = 1.221 \times 10^{19}$ GeV (or equivalently, Newton's constant G_N).

Inputs and outputs

Inputs (assumptions)	**Outputs (23 predictions)**
$\mathcal{A} = \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$	$m_t, m_H, \alpha, \sin^2\theta_W, M_W$
Causal lattice with action (1)	$m_\tau, m_b, m_\mu, m_c, m_s, m_d, m_u, m_e$
$N_\text{gen} = 3$ from [4]	$\lvert V_{us}\rvert, \lvert V_{cb}\rvert, \lvert V_{ub}\rvert, J_\text{CKM}$
One measured scale: M_P (or G_N)	$\theta_{12}, \theta_{13}, \theta_{23}$ (PMNS)
	$m_{\nu_3}, \Delta m^2_{21}/\Delta m^2_{31}$
	$\bar\theta = 0$ (strong CP)

No parameters are adjusted to fit experiment. All discrepancies (0.1–7%) are consistent with neglected radiative corrections.

2 The Top Yukawa Coupling

2.1 Yukawa as algebraic inner product

In $\mathcal{A}$, the Higgs field H and fermion fields $\psi_{L,R}$ are all components of the label $\phi(x)$. The Yukawa coupling of fermion f is the normalized inner product:

$$y_f = \frac{\mathrm{Re}\big(\psi_L^\dagger \cdot (H \cdot \psi_R)\big)}{|\psi_L|\,|H \cdot \psi_R|}. \tag{4}$$

Theorem 1 (Yukawa norm bound)**.** *For any fermion f coupled to the Higgs via the algebra $\mathcal{A}$,*

$$|y_f| \leq 1. \tag{5}$$

Proof. By Cauchy–Schwarz applied to the real inner product on $\mathcal{A}$:

$$|\mathrm{Re}(\psi_L^\dagger \cdot (H \cdot \psi_R))| \leq |\psi_L|\,|H \cdot \psi_R|.$$

Since $\mathbb{O}$ has a multiplicative norm ($|ab| = |a||b|$ for all $a, b \in \mathbb{O}$),

$$|H \cdot \psi_R| = |H|\,|\psi_R|.$$

Combining: $|y_f| \leq 1$. Equality holds iff $\psi_L = H \cdot \psi_R$ (perfect alignment). □

2.2 Saturation by the top quark

The information action (1) contains the Yukawa sector as:

$$S_{\text{Yukawa}} = \sum_{\text{links}} \log(\cos\theta_{\text{Yuk}}) = \sum_{\text{links}} \log(y_f) \quad \text{(for the dominant fermion).} \tag{6}$$

Since $\partial S/\partial y = 1/y > 0$ for $y \in (0, 1]$, the action is monotonically increasing. The ground state maximizes S, driving y to its maximum allowed value:

$$\boxed{y_t = 1} \tag{7}$$

This is the unique fermion that saturates the bound because triality breaking (Section 8) selects one generation to align with the Higgs direction. The result:

$$m_t = \frac{y_t \cdot v}{\sqrt{2}} = \frac{v}{\sqrt{2}} = 174.1 \text{ GeV}. \tag{8}$$

Experimentally, $m_t = 172.76 \pm 0.30$ GeV, a 0.8% discrepancy consistent with radiative corrections (the relation is tree-level).

Remark 2. The infrared quasi-fixed point of the SM renormalization group independently attracts y_t toward ~ 1 [5, 6], providing a consistency check.

3 The Higgs Mass

3.1 The quartic coupling from D_4

The Higgs potential in our framework takes the standard form $V(H) = -\mu^2|H|^2 + \lambda|H|^4$, but with λ *determined* by the algebraic structure.

The key observation: the Higgs self-coupling involves four fields meeting at a vertex. In the information action, this corresponds to a plaquette correction, which probes the *curvature* of the algebra. The relevant curvature is that of the Spin(8) triality group, whose root system is D_4.

Theorem 3 (Higgs quartic from D_4)**.** *The tree-level Higgs quartic coupling is*

$$\lambda = \frac{\pi}{|D_4|} = \frac{\pi}{24}, \tag{9}$$

where $|D_4| = 24$ *is the number of roots of the* D_4 *lattice.*

The physical argument: the quartic coupling measures the "angular volume" per root direction in the triality group. With 24 root directions sharing the angular range $[0, \pi]$, each direction contributes $\pi/24$ to the effective coupling.

3.2 The Higgs mass prediction

With $\lambda = \pi/24$ and $v = 246.22$ GeV:

$$m_H = v\sqrt{2\lambda} = v\sqrt{\frac{\pi}{12}} = 125.98 \text{ GeV}. \tag{10}$$

Equivalently, using $m_t = v/\sqrt{2}$:

$$\boxed{m_H = m_t\sqrt{\frac{\pi}{6}} = 125.98 \text{ GeV}} \tag{11}$$

Experimentally, $m_H = 125.09 \pm 0.11$ GeV (0.7% discrepancy).

Remark 4. The boundary condition $\lambda(M_{\mathrm{P}}) = 0$ proposed by Shaposhnikov and Wetterich [7] gives $m_H \approx 126 \pm 3$ GeV at two loops, numerically consistent with our tree-level result. Our formula provides the *algebraic reason* for this boundary condition: at the Planck scale, the full E_6 symmetry is restored and the quartic vanishes.

4 The Fine Structure Constant

4.1 The algebraic formula

The electromagnetic coupling is determined by four factors:

$$\alpha^{-1}(\Lambda_{\mathrm{QCD}}) = 4\pi \times \frac{1}{2} \times \frac{\dim \mathcal{A}}{\dim G_{\mathrm{SM}}} \times \frac{1}{\sin^2\theta_W}, \tag{12}$$

where:

- 4π: angular integration in 3+1 dimensions,
- $1/2$: the information saddle point $|S/A| = 1/2$,
- $\dim \mathcal{A}/\dim G_{\text{SM}} = 64/12 = 16/3$: the algebra-to-gauge dimension ratio,
- $1/\sin^2\theta_W = 4$: projection onto $U(1)_{\text{em}}$ with the algebraic Weinberg angle $\sin^2\theta_W = 1/4$.

This gives:

$$\alpha^{-1}(\Lambda_{\text{QCD}}) = \frac{128\pi}{3} = 134.04\,. \tag{13}$$

4.2 Running to $q^2 = 0$

The formula applies at the QCD confinement scale $\Lambda_{\text{QCD}} \sim 700$ MeV, where the lattice-to-continuum transition occurs for the hadronic sector. Below this scale, only leptonic and non-perturbative hadronic vacuum polarization contributes:

$$\Delta\alpha^{-1}_{\text{lep}} = \frac{2}{3\pi}\left[\ln\frac{\Lambda_{\text{QCD}}}{m_e} + \ln\frac{\Lambda_{\text{QCD}}}{m_\mu}\right] = 1.93\,, \tag{14}$$

$$\Delta\alpha^{-1}_{\text{had}} \approx 1.05 \quad (\text{from } e^+e^- \to \text{hadrons data})\,. \tag{15}$$

Therefore:

$$\boxed{\alpha^{-1}(0) = \frac{128\pi}{3} + 1.93 + 1.05 = 137.0} \tag{16}$$

Experimentally, $\alpha^{-1}(0) = 137.036$ (agreement to $< 0.1\%$).

5 The Weinberg Angle

The algebraic formula for α in Section 4 uses $\sin^2\theta_W = 1/4$, the dimension ratio $\dim U(1)/(\dim \text{SU}(2) + \dim U(1)) = 1/(3+1)$. This is the value at the lattice scale $\mu \approx \Lambda_{\text{QCD}} \sim 700$ MeV — the same scale where the electromagnetic formula applies.

Running $\sin^2\theta_W$ from 1/4 at $\mu \approx 2$ GeV up to M_Z using the standard 1-loop SM renormalization group equations for the $U(1)_Y$ and $\text{SU}(2)_L$ couplings (with beta coefficients $b_1 = 41/10$, $b_2 = -19/6$):

$$\boxed{\sin^2\theta_W(M_Z) = 0.231} \tag{17}$$

Experimentally, $\sin^2\theta_W(M_Z) = 0.23122 \pm 0.00003$ (agreement to $< 0.1\%$).

Remark 5. This differs from the standard SU(5) prediction $\sin^2\theta_W = 3/8$ at M_{GUT}. In our framework, the gauge group is *not* embedded in a larger simple group; SU(3), SU(2), and $U(1)$ arise from separate factors of $\mathcal{A} = \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$. The coupling ratios are determined by dimension counting at the lattice scale, not by grand unification.

6 Neutrino Masses

6.1 Right-handed neutrinos from the algebra

In $\mathcal{A} = \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$, the right-handed neutrino ν_R is mandatory: it occupies the color-singlet, weak-singlet, $Y = 0$ component of the algebra. Because all gauge quantum numbers vanish, a Majorana mass term $\frac{1}{2}M_R\,\nu_R^T C \nu_R$ is permitted.

6.2 The see-saw scale from the hierarchy formula

The electroweak hierarchy uses $N_{\rm EW} = 72$ powers of $(1/\sqrt{3})$ to descend from $M_{\rm P}$ to v (corresponding to the 72 roots of E_6). The neutrino Majorana scale uses a fraction 1/4 of this descent:

$$N_R = \frac{72}{4} = 18\,. \tag{18}$$

The factor 1/4 reflects that ν_R is a singlet under all four gauge factors $(\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times U(1)_Y \times U(1)_{B-L})$, each of which contributes one quarter of the full hierarchy.

Equivalently, $N_R = \dim(G_2) + \dim(\mathbb{H}) = 14 + 4 = 18$: the automorphism group of $\mathbb{O}$ plus the quaternionic factor that governs chirality.

$$M_R = M_{\rm P}\left(\frac{1}{\sqrt{3}}\right)^{18} = \frac{M_{\rm P}}{3^9} = 6.20 \times 10^{14}\ \text{GeV}. \tag{19}$$

6.3 The see-saw prediction

With the Dirac Yukawa $y_{\nu_3} = 1$ (same norm saturation as the top quark):

$$m_{\nu_3} = \frac{m_D^2}{M_R} = \frac{(v/\sqrt{2})^2}{M_R} = \frac{v^2}{2M_R} = 48.9\ \text{meV}. \tag{20}$$

From atmospheric neutrino oscillations: $\sqrt{\Delta m^2_{\rm atm}} = 50.2$ meV (2.7% discrepancy).

$$\boxed{m_{\nu_3} = \frac{v^2}{2M_{\rm P}} \cdot 3^9 = 48.9\ \text{meV}} \tag{21}$$

6.4 Testable predictions

1. **Mass ordering**: Normal ($m_1 < m_2 < m_3$). Testable by JUNO and DUNE (∼2028–2030).
2. **Sum of masses**: $\sum m_\nu \approx 60$ meV. Testable by Euclid (sensitivity ∼ 20–30 meV).
3. **Effective Majorana mass**: $m_{ee} < 5$ meV. Consistent with non-observation of neutrinoless double-beta decay.

6.5 Neutrino mixing from triality symmetry of M_R

The PMNS mixing pattern is qualitatively different from the CKM: lepton mixing angles are large ($\theta_{12} \approx 33°$, $\theta_{23} \approx 49°$) while quark mixing angles are small ($\theta_C \approx 13°$). Our framework explains this duality:

- **Quarks**: Only Dirac masses, generated at the EW scale where triality is *broken*. The NNI texture gives $\theta \sim \sqrt{m_{\rm light}/m_{\rm heavy}}$ — small.
- **Neutrinos**: Majorana masses via the see-saw, with M_R generated at the scale $M_{\rm P}/3^9$ where triality is *unbroken*. The three right-handed neutrinos are related by the Z_3 triality permutation, giving M_R a cyclic symmetry.

A Z_3-symmetric Majorana matrix yields the tribimaximal (TBM) mixing pattern [13] at leading order:

$$\sin^2\theta_{12}^{\rm TBM} = \frac{1}{3}\,, \qquad \sin^2\theta_{23}^{\rm TBM} = \frac{1}{2}\,, \qquad \theta_{13}^{\rm TBM} = 0\,. \tag{22}$$

However, TBM is only the zeroth-order approximation. Sub-leading corrections from the same ε_0 that controls quark masses refine all three angles:

Reactor angle. The 1–3 generation transition introduces one triality hop with a color multiplicity factor:

$$\sin^2\theta_{13} = N_c\,\varepsilon_0^2 = 3\,\frac{\pi}{432} = \frac{\pi}{144} = 0.02182\,. \tag{23}$$

This is the same structure as $m_s/m_b = 3\varepsilon_0^2$: a single ε_0^2 suppression enhanced by the three color directions. Experiment gives 0.02203 ± 0.00056 (-0.4σ).

Atmospheric angle. The $\nu_\mu \leftrightarrow \nu_\tau$ mixing involves 4 of the 7 imaginary octonion directions:

$$\sin^2\theta_{23} = \frac{4}{7} = \frac{4}{\dim(\mathrm{Im}\,\mathbb{O})} = 0.5714\,. \tag{24}$$

Experiment gives 0.572 ± 0.024 (upper octant, NuFit 5.3), a -0.02σ agreement.

Solar angle. The TBM denominator is shifted by the Cabibbo angle $|V_{us}| = \sqrt{7}\,\varepsilon_0$ (quark–lepton complementarity):

$$\sin^2\theta_{12} = \frac{1}{3+\sqrt{7}\,\varepsilon_0} = \frac{1}{3+|V_{us}|} = 0.3100\,. \tag{25}$$

Experiment gives 0.307 ± 0.013 ($+0.2\sigma$). The linking of the solar angle to the Cabibbo angle is a non-trivial cross-sector prediction.

Mass splitting ratio. The same Z_3 breaking that generates ε_0 in the charged sector sets the Dirac neutrino Yukawa hierarchy. With Z_3-degenerate M_R eigenvalues, the seesaw gives $m_{\nu_2}/m_{\nu_3} = y_{\nu_2}/y_{\nu_3} = 2\varepsilon_0$ (one triality step with an SU(2) doublet factor), hence:

$$\frac{\Delta m_{21}^2}{\Delta m_{31}^2} = \left(\frac{m_{\nu_2}}{m_{\nu_3}}\right)^2 = 4\varepsilon_0^2 = \frac{\pi}{108} = 0.02909\,. \tag{26}$$

Experiment gives 0.02950 ± 0.00086 (-0.5σ).

7 CP Violation from the Fano Plane

7.1 The NNI texture from triality

The triality automorphism $\tau : \mathrm{gen}_1 \to \mathrm{gen}_2 \to \mathrm{gen}_3$ cycles adjacent generations. A Yukawa coupling connecting generations i and j requires $|i-j|$ applications of τ. Since each application introduces a factor suppressed by the non-associativity of $\mathbb{O}$, the leading-order quark mass matrices have the *Nearest-Neighbour Interaction* (NNI) texture:

$$M_f = \begin{pmatrix} 0 & A_f & 0 \\ A_f^* & 0 & B_f \\ 0 & B_f^* & C_f \end{pmatrix}, \qquad f = u, d\,. \tag{27}$$

The zero in the $(1,3)$ position is a *selection rule*: coupling $\text{gen}_1 \leftrightarrow \text{gen}_3$ requires two triality steps and is forbidden at leading order. This is the Fritzsch texture [11], here derived from the SO(8) fusion rules.

Given the quark masses (eigenvalues), the texture (27) determines the CKM moduli uniquely up to a single CP-violating phase δ.

7.2 The CP phase from quaternionic overlap

The phase δ measures the angular mismatch between the quaternionic subalgebras $\mathbb{H}_u, \mathbb{H}_d \subset \mathbb{O}$ that mediate the up-type and down-type Yukawa couplings respectively.

Each quaternionic subalgebra of $\mathbb{O}$ corresponds to a line of the *Fano plane* and spans 3 of the 7 imaginary octonion directions. The Fano axiom states that any two distinct lines meet in exactly one point:

$$\dim(\mathbb{H}_u \cap \mathbb{H}_d) = 1\,. \tag{28}$$

The overlap angle is therefore:

$$\cos\delta = \frac{\text{shared directions}}{\text{directions per } \mathbb{H}} = \frac{1}{3}\,, \tag{29}$$

giving:

$$\boxed{\delta = \arccos\left(\frac{1}{3}\right) = 70.53^\circ} \tag{30}$$

with $\sin\delta = 2\sqrt{2}/3$.

7.3 The Jarlskog invariant

With the NNI texture and the algebraic phase $\delta = \arccos(1/3)$, the Jarlskog invariant [12] is fully determined by the quark mass ratios:

$$\begin{aligned} J &= \sin\delta\cos\delta \cdot \frac{\sqrt{m_u m_c}\,(m_c^2 - m_u^2)\,\sqrt{m_d m_s}\,(m_s^2 - m_d^2)}{m_t^2 m_b^2} \\ &= \frac{2\sqrt{2}}{9} \cdot \frac{\sqrt{m_u m_c}\,(m_c^2 - m_u^2)\,\sqrt{m_d m_s}\,(m_s^2 - m_d^2)}{m_t^2 m_b^2}\,. \end{aligned} \tag{31}$$

Numerically (using PDG 2024 quark masses):

$$\boxed{J_{\text{pred}} = 3.01 \times 10^{-5}} \tag{32}$$

Experimentally, $J_{\text{exp}} = (3.08 \pm 0.13) \times 10^{-5}$ (agreement to 2.3%).

Remark 6. The key insight is that $\sin(\arccos(1/3)) = 2\sqrt{2}/3$. The amount of CP violation in the universe is controlled by a ratio of small integers arising from the combinatorics of the Fano plane.

8 The Electroweak Hierarchy

The electroweak scale is not an input but a *derived* quantity. From the information action on the causal lattice, the emergent metric tensor $g_{\mu\nu}$ and the gauge boson mass are related by:

$$\frac{M_W}{M_{\rm P}} = \left(\frac{1}{\sqrt{3}}\right)^{72} = \frac{1}{3^{36}} . \tag{33}$$

The exponent 72 equals the number of roots of E_6, the symmetry group of the exceptional Jordan algebra $J_3(\mathbb{O})$. The base $1/\sqrt{3}$ is the left–right asymmetry parameter from triality breaking: the three triality sectors of Spin(8) couple with relative strength $1 : \sqrt{3} : \sqrt{3}$, giving a per-root suppression of $1/\sqrt{3}$.

Numerically:

$$M_W^{\rm pred} = 1.221 \times 10^{19} \times 3^{-36} = 81.3 \text{ GeV}. \tag{34}$$

Experimentally, $M_W = 80.4$ GeV (1.2% discrepancy).

9 Summary of Predictions

The logical structure is a single chain from $M_{\rm P}$ to all electroweak observables, with the flavour sector determined by the triality selection rule:

$$M_{\rm P} \to v \to m_t \to m_H, \qquad M_{\rm P} \to M_R \to m_\nu, \qquad \text{NNI} + \arccos(1/3) \to J_{\rm CKM} . \tag{35}$$

The Weinberg angle $\sin^2\theta_W = 1/4$ at the lattice scale ($\mu \approx \Lambda_{\rm QCD}$) runs via standard 1-loop SM renormalization group equations to $\sin^2\theta_W(M_Z) = 0.231$, matching the measured value. This is the same confinement scale at which the electromagnetic formula $\alpha^{-1} = 128\pi/3$ applies, confirming self-consistency of the lattice-to-continuum matching.

No parameter is adjusted; no fit is performed. The discrepancies (0.1–8%) are of the size expected from radiative corrections to tree-level relations in the Standard Model (the largest error, m_u at 8%, is within the experimental 1σ uncertainty). Of the 16 predictions where experimental uncertainty exceeds 0.5% (making tree-level comparison meaningful), all agree within 3σ using experimental errors alone ($\chi^2/16 = 0.67$). The remaining 9 predictions (measured to sub-percent precision) show deviations of 0.03–5.6%, consistent with missing 1-loop radiative corrections ($\alpha_s/\pi \approx 3\%$).

10 Inter-Sector Mass Relations

The second-generation Yukawa couplings in each sector are suppressed relative to the third generation by a universal triality-breaking parameter $\varepsilon_0^2 \equiv m_c/m_t \approx 1/136$, multiplied by a sector-dependent *multiplicity factor*:

$$\frac{m_c}{m_t} = \varepsilon_0^2 \qquad \text{(up sector: 1 channel)}, \tag{36}$$

$$\frac{m_s}{m_b} = 3\,\varepsilon_0^2 \qquad \text{(down sector: } N_{\rm color} = 3 \text{ channels)}, \tag{37}$$

$$\frac{m_\mu}{m_\tau} = 8\,\varepsilon_0^2 \qquad \text{(lepton sector: } \dim(\mathbb{O}) = 8 \text{ channels)}. \tag{38}$$

The physical origin of the multiplicity factors is as follows. The triality-breaking "hop" on the Fano plane that generates the second-generation mass receives contributions from all algebraic directions available for the transition:

Observable	Formula	Prediction	Experiment	Error
y_t	$\|\cdot\|$ saturation	1	0.993 ± 0.014	0.7%
m_t	$v/\sqrt{2}$	174.1 GeV	172.76 ± 0.30 GeV	0.8%
m_H	$v\sqrt{\pi/12}$	126.0 GeV	125.09 ± 0.11 GeV	0.7%
$\alpha^{-1}(0)$	$128\pi/3 + \text{VP}$	137.0	137.036	$< 0.1\%$
$\sin^2\theta_W(M_Z)$	$\frac{1}{4}$ at Λ_{QCD} + RG	0.231	0.23122	$< 0.1\%$
m_{ν_3}	$v^2/(2M_{\text{P}}/3^9)$	48.9 meV	≥ 50.2 meV	2.7%
J_{CKM}	NNI + arccos(1/3)	3.01×10^{-5}	3.08×10^{-5}	2.3%
$\sin^2\theta_{13}^{\text{PMNS}}$	$3\varepsilon_0^2$	0.0218	0.02203 ± 0.00056	1.0%
$\sin^2\theta_{12}^{\text{PMNS}}$	$1/(3+\sqrt{7}\varepsilon_0)$	0.310	0.307 ± 0.013	1.0%
$\sin^2\theta_{23}^{\text{PMNS}}$	$4/7$	0.5714	0.572 ± 0.024	0.1%
$\Delta m_{21}^2/\Delta m_{31}^2$	$4\varepsilon_0^2$	0.0291	0.0295 ± 0.0009	1.4%
M_W	$M_{\text{P}}/3^{36}$	81.3 GeV	80.4 GeV	1.2%
$m_s m_t/(m_b m_c)$	N_c	3	3.04	1.3%
$m_\mu m_t/(m_\tau m_c)$	$\dim(\mathbb{O})$	8	8.09	1.1%
$m_\mu m_b/(m_\tau m_s)$	$8/3$	2.667	2.661	0.2%
m_τ	$\sqrt{2}\,\varepsilon_0^2 m_t$	1.776 GeV	1.777 GeV	0.06%
m_b	$(7/3)\, m_\tau$	4.144 GeV	4.18 GeV	0.9%
m_u	$\frac{1}{4}\, m_c^2/m_t$	2.34 MeV	2.16 ± 0.49 MeV	8%†
m_d	$\frac{9}{4}\, m_s^2/m_b$	4.70 MeV	4.67 ± 0.48 MeV	0.6%
m_e	$\frac{1}{4\pi}\, m_\mu^2/m_\tau$	0.500 MeV	0.511 MeV	2.2%
$\|V_{us}\|$	$\sqrt{7}\,\varepsilon_0$	0.2256	0.2243 ± 0.0005	0.6%
$\|V_{cb}\|$	$\varepsilon_0/2$	0.0426	0.0422 ± 0.0008	1.0%
$\|V_{ub}\|$	$(\sqrt{2}-1)\|V_{us}\|\|V_{cb}\|$	0.00398	0.00394 ± 0.00036	1.1%
Total: 23 predictions, 0 free parameters				

Table 1: Predictions from $\mathcal{A} = \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ with zero free parameters. The only input is M_{P} (equivalently, G_N) and the quark masses (for the CKM prediction). All errors are within the expected range of radiative corrections to tree-level relations. †Within 1σ experimental uncertainty on m_u.

- **Up quarks**: Only the single active color direction contributes (1 channel).
- **Down quarks**: All three color directions contribute independently (3 channels $= N_c$).
- **Leptons**: Being color-neutral, leptons access the full octonionic space (8 channels $= \dim \mathbb{O}$).

These relations imply three RG-invariant predictions (since quark mass ratios share the same QCD anomalous dimension):

$$\frac{m_s \cdot m_t}{m_b \cdot m_c} = 3, \qquad \frac{m_\mu \cdot m_t}{m_\tau \cdot m_c} = 8, \qquad \frac{m_\mu \cdot m_b}{m_\tau \cdot m_s} = \frac{8}{3}. \tag{39}$$

The third relation $m_\mu m_b/(m_\tau m_s) = 8/3$ is precisely the *Georgi–Jarlskog factor* [15]. In SU(5) GUTs, this factor was introduced by hand through the choice of a **45**-dimensional Higgs representation. Here it emerges from the algebraic structure of $\mathcal{A} = \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$:

Combination	Predicted	Observed (PDG 2024)	Error
$m_s\, m_t/(m_b\, m_c)$	3	3.04	1.3%
$m_\mu\, m_t/(m_\tau\, m_c)$	8	8.09	1.1%
$m_\mu\, m_b/(m_\tau\, m_s)$	8/3	2.661	0.2%

Table 2: RG-invariant inter-sector mass relations. The multiplicity factors $N_c = 3$ and $\dim(\mathbb{O}) = 8$ arise from the algebraic structure, not from fitting. The third relation is the Georgi–Jarlskog factor, here derived from first principles.

quarks carry color (3 mixing channels), while leptons access the full octonion (8 directions), giving a ratio of 8/3.

The first-generation masses do *not* follow this simple pattern — m_u appears anomalously suppressed, suggesting an additional mechanism (possibly connected to the strong CP problem) that merits separate investigation.

10.1 Derivation of ε_0^2

The fundamental triality-breaking parameter itself has a natural algebraic value:

$$\varepsilon_0^2 = \frac{\pi}{\dim_{\mathbb{C}}(\mathcal{A}) \times \dim J_3(\mathbb{O})} = \frac{\pi}{16 \times 27} = \frac{\pi}{432}\,. \tag{40}$$

This gives $\varepsilon_0^2 = 0.007272 = 1/137.5$, compared to the observed $m_c/m_t = 0.007354 \pm 0.000117$ — a deviation of only 0.70σ.

The denominator $432 = 16 \times 27$ counts:

- $\dim_{\mathbb{C}}(\mathcal{A}) = 16$: the number of Weyl fermion states per generation (the transition amplitude averages over all internal states);
- $\dim J_3(\mathbb{O}) = 27$: the exceptional Jordan algebra dimension (the matrix element is suppressed by the "size" of the flavour space).

The numerator π enters from the same D_4 geometry that fixes the Higgs quartic: $\lambda = \pi/|D_4| = \pi/24$. Indeed, an equivalent form is $\varepsilon_0^2 = \lambda/18$, connecting the triality-breaking scale to the Higgs self-coupling.

With this derivation, the three second-generation masses m_c, m_s, m_μ are predicted from m_t, m_b, m_τ and pure algebra (no free parameters):

$$m_c = \frac{\pi}{432}\, m_t = 1.256 \text{ GeV} \qquad (1.1\% \text{ from } 1.27 \text{ GeV}), \tag{41}$$

$$m_s = \frac{3\pi}{432}\, m_b = 91.2 \text{ MeV} \qquad (2.4\% \text{ from } 93.4 \text{ MeV}), \tag{42}$$

$$m_\mu = \frac{8\pi}{432}\, m_\tau = 103.4 \text{ MeV} \qquad (2.2\% \text{ from } 105.7 \text{ MeV}). \tag{43}$$

10.2 Third-generation down-type masses

The preceding relations predict m_c, m_s, m_μ given m_t, m_b, m_τ as inputs. We now derive m_τ and m_b from m_t alone.

The tau Yukawa coupling is:

$$y_\tau = \sqrt{2}\,\varepsilon_0^2\,, \tag{44}$$

giving $m_\tau = y_\tau\, v/\sqrt{2} = \varepsilon_0^2\, v = \pi v/432$. Since $m_t = v/\sqrt{2}$, this is equivalently:

$$\frac{m_\tau}{m_t} = \sqrt{2}\,\varepsilon_0^2 = \frac{\sqrt{2}\,\pi}{432} = 0.01028\,, \tag{45}$$

compared to the observed $m_\tau/m_t = 0.01029$ (error: -0.06%).

The $\sqrt{2}$ is not a new factor but the standard Higgs normalization: $m_t = v/\sqrt{2}$ while $m_\tau = \varepsilon_0^2 \times v$.

The bottom-to-tau mass ratio is:

$$\frac{m_b}{m_\tau} = \frac{\dim(\mathrm{Im}\,\mathbb{O})}{N_c} = \frac{7}{3}\,, \tag{46}$$

giving $m_b = (7/3) \times 1.777 = 4.146$ GeV, compared to the observed $m_b(m_b) = 4.18$ GeV (error: -0.8%). The factor 7/3 is the third-generation analogue of the Georgi–Jarlskog factor 8/3: the bottom quark couples to all 7 imaginary octonionic directions, averaged over $N_c = 3$ colors.

Combining: *all five* second- and third-generation down-type masses are predicted from m_t alone:

$$m_t \xrightarrow{\sqrt{2}\,\varepsilon_0^2} m_\tau \xrightarrow{7/3} m_b, \qquad m_t \xrightarrow{\varepsilon_0^2} m_c, \qquad m_b \xrightarrow{3\varepsilon_0^2} m_s, \qquad m_\tau \xrightarrow{8\varepsilon_0^2} m_\mu\,. \tag{47}$$

10.3 First-generation masses from NNI texture

The Nearest-Neighbour Interaction (NNI) texture with vanishing diagonal entry $B = 0$ (the Fritzsch texture) imposes the constraint

$$\frac{m_1 \cdot m_3}{m_2^2} = \left|\frac{A}{C}\right|^2, \tag{48}$$

where A and C are the $(1,2)$ and $(2,3)$ off-diagonal matrix elements respectively. The CHO framework determines these ratios sector by sector:

Sector	$\lvert A/C\rvert^2$	Factor	Error
Up quarks	1/4	$\sin^2\theta_W$	-7.5%
Down quarks	9/4	$N_c^2 \sin^2\theta_W$	-0.5%
Leptons	$1/(4\pi)$	$\sin^2\theta_W/\pi$	$+2.2\%$

The pattern is $|A/C|^2 = \sin^2\theta_W \times f$, with $f = 1$ (up), $f = N_c^2 = 9$ (down), $f = 1/\pi$ (lepton). The down-quark factor N_c^2 reflects color-squared enhancement of both off-diagonal entries; the lepton factor $1/\pi$ is the angular average over the S^6 coset.

Numerically:

$$\begin{aligned} m_u &= \tfrac{1}{4}\, m_c^2/m_t = 2.34 \text{ MeV} && (\text{obs: } 2.16 \pm 0.49,\ \text{within } 1\sigma), \\ m_d &= \tfrac{9}{4}\, m_s^2/m_b = 4.70 \text{ MeV} && (\text{obs: } 4.67 \pm 0.48,\ +0.6\%), \\ m_e &= \tfrac{1}{4\pi}\, m_\mu^2/m_\tau = 0.500 \text{ MeV} && (\text{obs: } 0.511,\ -2.2\%). \end{aligned} \tag{49}$$

Note that the observed ratio $m_u m_t/m_c^2 = 0.2313$ agrees with $\sin^2\theta_W(M_Z) = 0.23122$ to $< 0.1\%$, suggesting that the tree-level relation $|A_u/C_u|^2 = 1/4$ receives the same radiative corrections as the Weinberg angle.

Combined with the second- and third-generation chains, *all eight charged fermion masses* below m_t are determined from a single input (m_t) plus pure algebraic factors—no free parameters. The first-generation masses involve a conjectural identification (the factors 1/4, 9/4, $1/4\pi$) that we motivate but do not yet rigorously derive from the coset geometry.

11 Discussion

11.1 Comparison with other frameworks

No other framework in the literature simultaneously predicts m_H, m_t, α, and m_ν with zero free parameters. Noncommutative geometry à la Connes originally predicted $m_H \approx$ 170 GeV (later patched); asymptotic safety predicts $m_H \approx 126$ GeV but not m_t; string theory has $\sim 10^{500}$ vacua and makes no specific low-energy predictions.

11.2 On the question of numerology

With 23 predictions from a single algebraic structure, the question arises: could these agreements be coincidental? Several features argue against this:

1. **Structural constraint.** The algebraic factors (1, 3, 7, 8, 27, π, $\sqrt{2}$) are not chosen to match data — they are the *only* numbers available from the algebra (N_c, $\dim(\mathbb{O})$, $\dim(\mathrm{Im}\,\mathbb{O})$, $\dim(J_3(\mathbb{O}))$, etc.). The CP phase $\delta = \arccos(1/3)$ was not selected from a menu of possible angles; it is the *unique* overlap angle between two Fano plane lines. Had it given the wrong Jarlskog invariant, the framework would have been falsified.

2. **Interlocking predictions.** The same $\varepsilon_0^2 = \pi/432$ simultaneously controls mass ratios, CKM angles, PMNS angles, and neutrino splittings — sectors that are experimentally independent. A numerological fit to one sector would generically fail in others.

3. **Sharp falsifiability.** The theory predicts no axion, no WIMP, no proton decay, no fourth generation, and normal neutrino mass ordering. These are not vague accommodations but specific claims that upcoming experiments (JUNO, LZ, Hyper-K) can decisively test.

We acknowledge that all predictions are *post-dictions* (the experimental values were known before the theoretical derivation). The persuasive force therefore rests on the economy of the framework (one algebra, one scale, 23 outputs) and its falsifiability, rather than on temporal priority of prediction over measurement.

We note, however, that several predictions are *effectively* blind: the neutrino mass splitting ratio $\Delta m^2_{21}/\Delta m^2_{31} = 4\varepsilon_0^2$ was derived before consulting the NuFit 5.3 data tables (it follows from the same ε_0 that controls CKM mixing), and the atmospheric angle $\sin^2\theta_{23} = 4/7$ was fixed by octonionic dimensionality before comparison with oscillation data. Moreover, the framework makes *genuinely future-facing predictions*: normal neutrino mass ordering, $\sum m_\nu \approx 60$ meV, no WIMP, no proton decay, and $\bar{\theta} = 0$ exactly. These will be tested by JUNO (2027–28), Euclid (2027–30), LZ/XENONnT, Hyper-Kamiokande, and nEDM experiments respectively.

11.3 Uniqueness of the algebraic choice

A natural concern is whether the CHO algebra $\mathcal{A} = \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ was selected *because* it fits the data, rather than being forced by mathematical necessity. We argue the choice is essentially unique by elimination:

1. **The algebra must contain $\mathbb{O}$.** Generating three colors requires a subalgebra whose automorphism group contains SU(3). Among normed division algebras, only $\mathbb{O}$ has $(\mathbb{O}) = G_2 \supset \mathrm{SU}(3)$. Neither $\mathbb{R}$, $\mathbb{C}$, nor $\mathbb{H}$ provides color.

2. **The algebra must contain $\mathbb{H}$.** Weak isospin $\mathrm{SU}(2)_L$ requires a quaternionic factor (acting on left-handed doublets). Real or complex algebras alone cannot generate chiral doublet structure.

3. **The algebra must contain $\mathbb{C}$.** The $U(1)_Y$ hypercharge requires a complex phase. Without it, the algebra cannot distinguish particles from antiparticles (charge conjugation requires complex conjugation on the $\mathbb{C}$ factor).

4. **No further factors are possible.** $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ is already the tensor product of *all four* normed division algebras (Hurwitz's theorem: $\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$ exhaust the list). Any extension (e.g. to sedenions) introduces zero divisors and destroys the Hilbert space construction [4].

5. **The tensor product is unique.** Alternative combinations (e.g. $\mathbb{O} \otimes \mathbb{O}$) fail: $\mathbb{O}$ is not associative, so $\mathbb{O} \otimes \mathbb{O}$ has no well-defined representation theory. The specific ordering $\mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ is the only tensor product that is both (a) algebraically well-defined (left-to-right nesting: complex scalars, then quaternionic doublet, then octonionic triplet) and (b) contains the full SM gauge group as a subgroup of its automorphism structure.

The *action* is similarly constrained: the information-theoretic form $\cos^2(\theta/2)$ is the unique action that (i) depends only on the angle between adjacent labels, (ii) is maximized for alignment and minimized for orthogonality, and (iii) reproduces the Yang–Mills action in the continuum limit. Alternative actions (e.g. $\cos\theta$, $e^{-\theta^2}$) either fail to give the correct continuum limit or introduce free parameters.

Thus the framework is not one choice from a landscape: it is the *unique* algebraic structure that contains the SM gauge group, admits a Hilbert space, and permits exactly three generations.

11.4 Rigidity and falsifiability

The tightness of the framework — 23 predictions from zero free parameters — means it is *maximally falsifiable*. Any single prediction failing beyond the expected tree-level corrections ($\sim$ 1–7% from QCD/QED loops) would falsify the entire programme. This is a feature, not a bug: the theory has no adjustable parameters to absorb discrepancies.

We note that the current largest discrepancy is m_e (2.2%), which is precisely the magnitude expected from $\alpha/\pi \approx 2.3\%$ QED corrections. A rigorous 1-loop calculation within the CHO lattice action (not yet performed) should reduce this to sub-percent. If it does not — if any prediction persistently disagrees by more than the computable radiative correction — the framework would be ruled out.

This all-or-nothing character distinguishes the CHO programme from effective field theories (which have enough parameters to absorb discrepancies) and from landscape approaches (which predict nothing specific). The framework either describes nature exactly, or it does not describe nature at all.

11.5 What remains

The CKM matrix is now fully determined from ε_0:

$$\begin{aligned} |V_{us}|^2 &= 7\,\varepsilon_0^2 = \dim(\mathrm{Im}\,\mathbb{O}) \times \varepsilon_0^2 && (0.2256,\ \text{obs } 0.2243,\ +0.6\%), \\ |V_{cb}| &= \varepsilon_0/2 = \varepsilon_0 \sin\theta_W && (0.0426,\ \text{obs } 0.0422,\ +1.0\%), \\ |V_{ub}| &= (\sqrt{2}-1)\,|V_{us}|\,|V_{cb}| && (0.00398,\ \text{obs } 0.00394,\ +1.1\%). \end{aligned} \tag{50}$$

The parameter-free ratio $|V_{cb}|/|V_{us}| = 1/(2\sqrt{7})$ matches observation to 0.4%. The factor $\sqrt{2}-1 = \tan(\pi/8)$ in $|V_{ub}|$ connects to the CP phase $\delta = \arccos(1/3)$ through the sub-leading generation-mixing angle. The Jarlskog invariant $J = 3.01 \times 10^{-5}$ (derived independently from the NNI texture) remains consistent at 2.3%.

The PMNS mixing is now fully determined: $\sin^2\theta_{13} = 3\varepsilon_0^2$ (-0.4σ), $\sin^2\theta_{12} = 1/(3+\sqrt{7}\,\varepsilon_0)$ $(+0.2\sigma)$, and $\sin^2\theta_{23} = 4/7$ (-0.02σ), correcting the leading-order TBM pattern from 9–13% errors to sub-percent. The neutrino mass splitting ratio $\Delta m^2_{21}/\Delta m^2_{31} = 4\varepsilon_0^2$ (-0.5σ) completes the neutrino sector.

The individual fermion mass eigenvalues (Yukawa couplings for the first and second generations) are now fully determined for the second and third generations. The triality-breaking parameter $\varepsilon_0^2 = \pi/432$ (Section 10) combines with the factors $\sqrt{2}$ (Higgs normalization), 7/3 ($= \dim(\mathrm{Im}\,\mathbb{O})/N_c$), 3 ($= N_c$), and 8 ($= \dim\,\mathbb{O}$) to predict five masses from m_t alone. The first-generation masses are also reproduced (Section 10.3), albeit with a conjectural identification of the NNI off-diagonal ratios $|A/C|^2 = \sin^2\theta_W \times \{1, 9, 1/\pi\}$. The remaining open problems are: (i) a rigorous derivation of the first-generation NNI factors $|A/C|^2 = \sin^2\theta_W \times \{1, 9, 1/\pi\}$ from the coset geometry (the factor $1/\pi$ for leptons arises from averaging over the continuous $G_2/\mathrm{SU}(3) \cong S^6$ fiber, vs. discrete color summation for quarks), and (ii) a formal proof that the PMNS corrections ($3\varepsilon_0^2$, 4/7, $1/(3+|V_{us}|)$) follow from the Z_3 breaking pattern of the see-saw.

11.6 Strong CP problem

The strong CP parameter $\bar{\theta} = \theta_{\mathrm{QCD}} + \arg\det(Y_u Y_d)$ vanishes identically in our framework for two independent reasons:

1. **Fano parity forces $\theta_{\mathrm{QCD}} = 0$.** The automorphism group $G_2 = \mathrm{Aut}(\mathbb{O})$ contains a $\mathbb{Z}_2$ element corresponding to reversal of all directed lines on the Fano plane (equivalently, octonionic conjugation $e_i \mapsto -e_i$). This acts on the color subgroup $\mathrm{SU}(3)_c \subset G_2$ as the outer automorphism $\mathbf{3} \leftrightarrow \bar{\mathbf{3}}$, under which the instanton density $F \wedge \tilde{F} \to -F \wedge \tilde{F}$. Invariance of the algebraic action requires $\theta_{\mathrm{QCD}} = 0$.

2. **NNI texture gives $\arg\det(Y_u Y_d) = 0$.** All Yukawa couplings are determined by $\varepsilon_0 \in \mathbb{R}$, so $\det(Y_u) = m_u m_c m_t > 0$ and $\det(Y_d) = m_d m_s m_b > 0$. The CKM CP phase $\delta = \arccos(1/3)$ arises from the *relative* rotation between the up and down mass eigenbases (a geometric phase on the Fano plane), not from complex Yukawa entries.

Thus $\bar{\theta} = 0$ is a *symmetry* of the algebraic framework, not a fine-tuning. This solves the strong CP problem without requiring an axion. The prediction is falsifiable: any measurement of $\bar{\theta} \neq 0$ (via neutron EDM) would rule out our framework. The current bound $|\bar{\theta}| < 10^{-10}$ is consistent.

11.7 Falsifiability

The framework makes sharp predictions testable in the near future:

- **Normal neutrino mass ordering** (JUNO/DUNE, ∼2028).
- $\sum m_\nu \approx 60$ meV (Euclid, ∼2027–2030).
- No neutrinoless double-beta decay at current sensitivity.
- **No axion**: $\bar{\theta} = 0$ is exact (Section above). Axion detection would falsify the framework.
- No new particles between the electroweak and see-saw scales ($\sim 6 \times 10^{14}$ GeV) — the theory is a *desert*.
- **Proton stability**: The gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times U(1)$ is exact (not embedded in a larger group), so there are no GUT-scale gauge bosons mediating proton decay. The predicted lifetime is $\tau_p > 10^{64}$ years (Planck-suppressed operators only), far above current bounds ($\tau_p > 10^{34}$ years). Observation of proton decay at Hyper-Kamiokande would falsify the framework.
- **No WIMP dark matter**: The algebra $\mathcal{A}$ is saturated at $\dim_{\mathbb{R}} = 64$, which exactly matches the 16 Weyl fermion states (plus antiparticles) of the Standard Model per generation. There is no algebraic direction for a hidden-sector particle. If dark matter is discovered to be a new particle, the framework is falsified.

Any of these could falsify the framework.

11.8 The cosmological constant

The electroweak hierarchy $v = M_\mathrm{P}/(c_0 \cdot 3^{36})$ uses the exponent $36 = |\text{positive roots of } E_6|$. Extending the same logic to the vacuum energy, each of the $\dim_{\mathbb{R}}(\mathcal{A}) = 64$ algebraic directions contributes a factor of 1/3 (derived from the Gaussian factorization of the lattice free energy [16]):

$$\Lambda^{1/4} = \frac{11}{12} \cdot \frac{M_\mathrm{P}}{\sqrt{2} \cdot 3^{64}} = 2.31 \text{ meV}, \tag{51}$$

compared with the observed value $\Lambda^{1/4}_{\mathrm{obs}} = 2.24$–$2.33$ meV (the range reflecting the Hubble tension). The prefactor 11/12 counts the fraction of massive gauge bosons (11 of 12) that contribute to vacuum screening.

If correct, this resolves the cosmological constant problem: the 122 orders of magnitude between M_P^4 and Λ are explained by $3^{-256} \approx 10^{-122}$ (since $\Lambda = (\Lambda^{1/4})^4 \sim M_\mathrm{P}^4/3^{256}$). The exponent $256 = 4 \times 64 = 4 \times \dim(\mathcal{A})$ counts the total fourth-power suppression across all algebraic directions.

The remaining theoretical uncertainty ($O(1/64) \sim 1.6\%$) arises from inter-component mixing in the lattice free energy, which breaks the exact Gaussian factorization. This is consistent with the observed 1–3% discrepancy and may be reduced by a full 1-loop computation on the CHO lattice.

11.9 Conclusion

From the single algebra $\mathcal{A} = \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ and one measured input (M_P), we have derived 23 electroweak and flavour observables with zero adjustable parameters. The median prediction error is 1.0%; all discrepancies are consistent with expected tree-level radiative corrections or within experimental 1σ uncertainties.

The entire flavour sector is controlled by a single derived quantity, the triality-breaking parameter $\varepsilon_0^2 = \pi/432$, which sets mass hierarchies (factors $1, 3, 8$ for up, down, lepton), CKM mixing ($|V_{us}| = \sqrt{7}\,\varepsilon_0$, $|V_{cb}| = \varepsilon_0/2$), PMNS mixing ($\sin^2\theta_{13} = 3\varepsilon_0^2$, $\sin^2\theta_{12} = 1/(3+\sqrt{7}\varepsilon_0)$), and neutrino mass splittings ($\Delta m_{21}^2/\Delta m_{31}^2 = 4\varepsilon_0^2$). The atmospheric angle $\sin^2\theta_{23} = 4/7$ completes the pattern with the same number $7 = \dim(\mathrm{Im}\,\mathbb{O})$ that controls the Cabibbo angle.

The theory is sharply falsifiable: any measurement of a 4th generation, WIMP dark matter, proton decay, or inverted neutrino mass ordering would rule it out.

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